High School – Building Functions

Essential Questions:

How do you use patterns to understand mathematics and model situations?
What is algebra?
How are the horizontal and vertical axes related?
How do algebraic representations relate and compare to one another?
How can we communicate and generalize algebraic relationships?

Essential Vocabulary – functions, properties of functions, arithmetic sequences, geometric sequences, recursive formula, explicit formula, transformations, inverse of a function, composition of functions, invertible functions, non-invertible functions, domain, inverse relationships, exponents, logarithms

HS.F-BF.1: Write a function that describes a relationship between two quantities.
a. Determine an explicit expression, a recursive process, or steps for calculation from a context.
b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*
c. (+) Compose functions. *For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.*

Alg 3 Enduring Understandings

Students will know…	***Students will understand…***	***Students will be able to…***
1. function, explicit expression, recursive process, standard functions	1. that there exists a relationship between two quantities	1. write a function that describes a relationship between two quantities 2. combine functions using arithmetic operations 3. compose functions

HS.F-BF.2: Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations from a variety of contexts (e.g., science, history, and culture, including those of the Montana American Indian), and translate between the two forms.*

Alg 3 Enduring Understandings

Students will know…	***Students will understand…***	***Students will be able to…***
1. arithmetic sequences, geometric sequences, recursive formula, explicit formula	1. that arithmetic and geometric sequences are used in the real world	1. write sequences, use them to model situations, and translate between recursive and explicit formula

HS.F-BF.3: Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*

Alg 3 Enduring Understandings

Students will know…	***Students will understand…***	***Students will be able to…***
1. transformations, even functions, odd functions	1. that the graph of a function changes when the equation of a function is changed and the reasons for the changes	1. identify the effect that adding or multiplying a constant has on a graph 2. illustrate the change using technology 3. translate a graph horizontally $f(x + k)$ or vertically $f(x) + k$, by changing k,

HS.F-BF.4: Find inverse functions.

a. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. *For example, $f(x) = 2x^3$ or $f(x) = (x+1)/(x–1)$ for $x \neq 1$.*

b. (+) Verify by composition that one function is the inverse of another.

c. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse.

d. (+) Produce an invertible function from a non-invertible function by restricting the domain.

Alg 3 Enduring Understandings

Students will know…	***Students will understand…***	***Students will be able to…***
1. inverse of a function, composition of functions, invertible functions, non-invertible functions, domain	1. that not all functions have inverses	1. calculate the inverse of a function 2. verify two functions are inverses of each other by using compositions 3. read values of an inverse from a graph or a table, if the inverse exists 4. produce an invertible function from a non-invertible function by restricting the domain

HS.F-BF.5: (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

Alg 2 Enduring Understandings

Students will know…	***Students will understand…***	***Students will be able to…***
1. exponents, logarithms, inverse relationships between exponents and logarithms	1. that logarithmic functions and exponential functions are inverses	1. solve problems involving logarithms and exponents

High School – Interpreting Functions

Essential Questions:

How do you use patterns to understand mathematics and model situations?
What is algebra?
How are the horizontal and vertical axes related?
How do algebraic representations relate and compare to one another?
How can we communicate and generalize algebraic relationships?

Essential Vocabulary – function, domain, range, element, output, input, corresponding, graph, input, domain, output, function notation, sequence, function, recursive, subset, domain, integers, function, intervals, increasing, decreasing, maximums, minimums, symmetries, end behavior, periodicity, domain, function, graph, quantitative relationship, calculate, interpret, interval, rate of change, key features, linear, quadratic, intercepts, maxima, minima, square root, cube root, piece-wise functions, step functions, absolute value, end behavior, rational functions, asymptotes, exponential, logarithmic, trigonometric, period, midline, amplitude, equivalent forms, factoring, completing the square, extreme values, symmetry, exponents, exponential functions, properties of functions

HS.F-IF.1: Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.

Alg 1 Enduring Understandings

Students will know…	***Students will understand…***	***Students will be able to…***
1. domain, range, input, output, element	1. that for an equation to be a function each element of the domain corresponds to exactly one element in the range	1. determine if an equation is a function using domain and range

HS.F-IF.2: Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

Alg 1 Enduring Understandings

Students will know…	***Students will understand…***	***Students will be able to…***
1. functions, function notation, evaluate, input, domain, output	1. that function notation relates to various real world examples	1. interpret and evaluate statements in function notation

HS.F-IF.3: Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by $f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n-1)$ for $n \geq 1$.*

Alg 3 Enduring Understandings

Students will know…	***Students will understand…***	***Students will be able to…***
1. sequence, function, recursive, subset, domain, integers	1. that sequences are formed in different ways	1. recognize sequences as functions

HS.F-IF.4: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*

Alg 3 Enduring Understandings

Students will know…	***Students will understand…***	***Students will be able to…***
1. function, intervals, increasing, decreasing, maximums, minimums, symmetries, end behavior, periodicity	1. that key features of a function can be found on the graph of a function	1. interpret key features of graphs and tables of functions 2. sketch graph of functions using key features

HS.F-IF.5: Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*

Alg 2 Enduring Understandings

Students will know…	***Students will understand…***	***Students will be able to…***
1. domain, function, graph, quantitative relationship	1. that domain restriction is appropriate for a particular function given the context of the function	1. describe the domain of the function as it relates to the problem

HS.F-IF.6: Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

Alg 3 Enduring Understandings

Students will know…	***Students will understand…***	***Students will be able to…***
1. calculate, interpret, interval, rate of change	1. that the rate of change of a function is equivalent to the slope of the function at a given point or a given interval	1. calculate the average rate of change of a function 2. interpret the average rate of change of a function

HS.F-IF.7: Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

a. Graph linear and quadratic functions and show intercepts, maxima, and minima.

b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.

d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.

e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

Alg 3 Enduring Understandings		
Students will know…	***Students will understand…***	***Students will be able to…***
1. key features, linear, quadratic, intercepts, maxima, minima, square root, cube root, piece-wise functions, step functions, absolute value, end behavior, rational functions, asymptotes, exponential, logarithmic, trigonometric, period, midline, amplitude	1. that key features of various functions can be located by using the graph of the function	1. graph various functions using key features, by hand and with technology

HS.F-IF.8: Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
b. Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as* $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, *and classify them as representing exponential growth or decay.*

Alg 3, Alg 2 Enduring Understandings		
Students will know…	***Students will understand…***	***Students will be able to…***
1. function, equivalent forms, factoring, completing the square, extreme values, symmetry, exponents, exponential functions	1. that factoring, completing the square and properties of exponents can be used to rewrite functions	1. factor, complete the square and use properties of exponents to obtain equivalent forms of the function 2. write a function in different forms to identify key features

HS.F-IF.9: Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

Alg 1, Alg 2 Enduring Understandings		
Students will know…	***Students will understand…***	***Students will be able to…***
1. functions, properties of functions	1. that there are different ways to represent functions; including algebraically, graphically, numerically, and verbally	1. compare properties of functions

High School – Linear, Quadratic, and Exponential Models

Essential Questions:

How do you use patterns to understand mathematics and model situations?
What is algebra?
How are the horizontal and vertical axes related?
How do algebraic representations relate and compare to one another?
How can we communicate and generalize algebraic relationships?

Essential Vocabulary – linear functions, exponential functions, interval, growth, decay, linear functions, exponential functions, arithmetic and geometric sequences, input-output pair, graphs, tables, linear, quadratic, exponential, exponential form, logarithmic form, base, linear and exponential functions, parameters

HS.F-LE.1: Distinguish between situations that can be modeled with linear functions and with exponential functions.

a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.

b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

Alg 2 Enduring Understandings

Students will know…	***Students will understand…***	***Students will be able to…***
1. linear functions, exponential functions, interval, growth, decay	1. the difference between linear and exponential functions	1. determine when to use a linear function and when to use an exponential function 2. recognize growth and decay in functions

HS.F-LE.2: Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

Alg 3 Enduring Understandings

Students will know…	***Students will understand…***	***Students will be able to…***
1. linear functions, exponential functions, arithmetic and geometric sequences, input-output pair	1. that a graph, table and description are related to a specific function	1. construct a function given a graph, a table, or a description of a relationship

HS.F-LE.3: Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

Alg 1, Alg 2 Enduring Understandings

Students will know…	***Students will understand…***	***Students will be able to…***
1. graphs, tables, linear, quadratic, exponential	1. that functions can increase at different rates	1. observe that exponential functions will eventually exceed any polynomial functions

HS.F-LE.4: For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.

Alg 2 Enduring Understandings

Students will know…	***Students will understand…***	***Students will be able to…***
1. exponential form, logarithmic form, base	1. that exponents can be written as logarithms and logarithms can be written as exponents	1. rewrite exponentials as logarithmic 2. evaluate logarithms using technology

HS.F-LE.5: Interpret the parameters in a linear or exponential function in terms of a context.

Alg 2 Enduring Understandings

Students will know…	***Students will understand…***	***Students will be able to…***
1. linear and exponential functions, parameters	1. that real world problems have additional or different limitations depending on the context of the problem	1. interpret the parameters of linear and exponential functions in context

High School – Trigonometric Functions

Essential Questions:

How do you use patterns to understand mathematics and model situations?
What is algebra?
How are the horizontal and vertical axes related?
How do algebraic representations relate and compare to one another?
How can we communicate and generalize algebraic relationships?

Essential Vocabulary -

HS.F-TF.1: Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.

Alg 2/Trig Enduring Understandings

Students will know…	***Students will understand…***	***Students will be able to…***
1. the definition of a radian.	1. that the measure of an angle in radians is the length of the subtended arc.	1. find the measure of an angle in radians.

HS.F-TF.2: Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

Alg 2/Trig Enduring Understandings

Students will know…	***Students will understand…***	***Students will be able to…***
1. the definition of a unit circle and the location on the unit circle of radian measure	1. that radian measure is a real number.	1. calculate the trigonometric functions of all real numbers using the unit circle.

HS.F-TF.3: (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for $\pi - x$, $\pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number.

Alg 2/Trig Enduring Understandings

Students will know…	***Students will understand…***	***Students will be able to…***
1. special right triangles and the relationships of their sides and the properties of the unit circle.	1. that the relationships of the trigonometric functions as you rotate through the quadrants as well as the special angles in each quadrant are the same except for a change in sign	1. find the sine, cosine and tangent of $\pi/3$, $\pi/4$ and $\pi/6$ geometrically and x, any real number, using the unit circle and its properties.

HS.F-TF.4: (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.		
Algebra 3 Enduring Understandings		
Students will know…	***Students will understand…***	***Students will be able to…***
1. the definition of symmetry in regards to even and odd functions and periodicity.	1. that even and odd functions are defined due to their symmetry.	1. describe trigonometric functions as being even or odd.
HS.F-TF.5: Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.*		
Algebra 3 Enduring Understandings		
Students will know…	***Students will understand…***	***Students will be able to…***
1. the definitions of amplitude, frequency and midline.	1. that some real world phenomena that is cyclical by nature may be modeled with trigonometric functions.	1. choose the correct trigonometric function to model cyclical behavior
HS.F-TF.6: (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.		
Alg 2/Trig Enduring Understandings		
Students will know…	***Students will understand…***	***Students will be able to…***
1. the definition of inverse.	1. that in order to find the inverse of a trigonometric function the domain must be limited.	1. find the inverse of a trigonometric function.
HS.F-TF.7: (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.*		
Alg 2/Trig Enduring Understandings		
Students will know…	***Students will understand…***	***Students will be able to…***
1. inverse trigonometric functions.	1. that inverse functions are needed to find solutions to some modeling problems.	1. solve trigonometric equations using inverse functions 2. evaluate solutions using appropriate technology 3. interpret solutions in context

HS.F-TF.8: Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle.		
Alg 2/Trig Enduring Understandings		
Students will know…	***Students will understand…***	***Students will be able to…***
1. the Pythagorean Theorem.	1. that the Pythagorean Theorem and the unit circle are used to develop the identity $\sin^2(\theta) + \cos^2(\theta) = 1$.	1. use the Pythagorean Theorem to prove the Pythagorean Identity, $\sin^2(\theta) + \cos^2(\theta) = 1$, and to solve problems using this identity.
HS.F-TF.9: (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.		
Geometry Enduring Understandings		
Students will know…	***Students will understand…***	***Students will be able to…***
1. the sum and difference formulas for sine, cosine and tangent.	1. that the proof of the addition and subtraction identities are based off the unit circle.	1. prove the addition and subtraction formulas for sine, cosine and tangent and then use them to solve problems.