

High School – Arithmetic with Polynomials and Rational Expressions

Essential Questions:

- How do you use patterns to understand mathematics and model situations?
- What is algebra?
- How are the horizontal and vertical axes related?
- How do algebraic representations relate and compare to one another?
- How can we communicate and generalize algebraic relationships?

Essential Vocabulary - polynomials, integers, closed under an operation, remainder theorem, zeros of a polynomial, function, polynomials identities

HS.A-APR.1: Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

Alg 1 Enduring Understandings

Students will know...

1. polynomials, integers, and the definition of closure

Students will understand...

1. that polynomials are closed under the operations of addition, subtraction and multiplication

Students will be able to...

1. add, subtract, and multiply polynomials

HS.A-APR.2: Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a , the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.

Alg 2 Enduring Understandings

Students will know...

1. the Remainder Theorem

Students will understand...

1. that the remainder when dividing a polynomial function by $(x - a)$ is $p(a)$

Students will be able to...

1. apply the remainder theorem when dividing polynomial functions

HS.A-APR.3: Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

Alg 3 Enduring Understandings

Students will know...

1. the definition of zeros of a polynomial function

Students will understand...

1. that the zeros of a polynomial function are the x -intercepts of the graph

Students will be able to...

1. identify zeros and construct a rough graph of the polynomial

<p>HS.A-APR.4: Prove polynomial identities and use them to describe numerical relationships. <i>For example, the polynomial identity $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples.</i></p>		
<p>Geometry and Algebra 1 Enduring Understandings</p>		
<p><i>Students will know...</i></p> <p>1. the definition of a polynomial identity</p>	<p><i>Students will understand...</i></p> <p>1. that using polynomial identities can describe numerical relationships</p>	<p><i>Students will be able to...</i></p> <p>1. prove polynomial identities</p>
<p>HS.A-APR.5: (+) Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.</p>		
<p>Alg 3 Enduring Understandings</p>		
<p><i>Students will know...</i></p> <p>1. the Binomial Theorem, Pascal's Triangle, and the meaning of variables and factorials</p>	<p><i>Students will understand...</i></p> <p>1. the relationship between the Binomial Theorem and Pascal's Triangle</p>	<p><i>Students will be able to...</i></p> <p>1. expand $(x + y)^n$</p>
<p>HS.A-APR.6: Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.</p>		
<p>Alg 2 Enduring Understandings</p>		
<p><i>Students will know...</i></p> <p>1. the definition of rational expressions and that you can divide a polynomial by a polynomial</p>	<p><i>Students will understand...</i></p> <p>1. there are various forms of rational expression and that they can be simplified using different methods</p>	<p><i>Students will be able to...</i></p> <p>1. simplify or rewrite rational expressions</p>
<p>HS.A-APR.7: (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.</p>		
<p>Alg 2 Enduring Understandings</p>		
<p><i>Students will know...</i></p> <p>1. definition of a rational expression and the definition of closure</p>	<p><i>Students will understand...</i></p> <p>1. that rational expressions are closed under addition, subtraction, multiplication and division because the rational numbers are closed under those operations</p>	<p><i>Students will be able to...</i></p> <p>1. add, subtract, multiply, and divide rational expressions</p>

High School – Creating Equations

Essential Questions:

- How do you use patterns to understand mathematics and model situations?
- What is algebra?
- How are the horizontal and vertical axes related?
- How do algebraic representations relate and compare to one another?
- How can we communicate and generalize algebraic relationships?

Essential Vocabulary - equations, inequalities, variables, linear functions, rational functions, exponential functions, quadratic functions, coordinate plane, constraints, formula

HS.A-CED.1: Create equations and inequalities in one variable and use them to solve problems from a variety of contexts (e.g., science, history, and culture), including those of Montana American Indians. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*

Alg 2 Enduring Understandings

<p><i>Students will know...</i></p> <p>1. the definition of equations, inequalities, variables, linear functions, rational functions, exponential functions, quadratic functions (absolute value)</p>	<p><i>Students will understand...</i></p> <p>1. that equations and inequalities can be used to model real life applications</p>	<p><i>Students will be able to...</i></p> <p>1. create and apply equations and inequalities to appropriate situations</p>
---	---	---

HS.A-CED.2: Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

Alg 1 Enduring Understandings

<p><i>Students will know...</i></p> <p>1. the definition of variables and the coordinate plane</p>	<p><i>Students will understand...</i></p> <p>1. the relationships between equations and graphs</p>	<p><i>Students will be able to...</i></p> <p>1. create and graph equations</p>
--	--	--

HS.A-CED.3: Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*

Alg 2 Enduring Understandings

<p><i>Students will know...</i></p> <p>1. the definition of constraints</p>	<p><i>Students will understand...</i></p> <p>1. the difference between a viable and nonviable solution</p>	<p><i>Students will be able to...</i></p> <p>1. interpret solutions based on the graph of constraints</p>
---	--	---

HS.A-CED.4: Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law $V = IR$ to highlight resistance R .*

Alg 1 Enduring Understandings

Students will know...

1. the definition of a formula

Students will understand...

1. the purpose of solving a formula for a specific variable

Students will be able to...

1. algebraic operations to solve a formula for a specific variable

DRAFT

High School – Reasoning with Equations and Inequalities

Essential Questions:

How do you use patterns to understand mathematics and model situations?

What is algebra?

How are the horizontal and vertical axes related?

How do algebraic representations relate and compare to one another?

How can we communicate and generalize algebraic relationships?

Essential Vocabulary - equations, solutions, rational equations, radical equations, extraneous solution, linear equations, coefficient, inequality, variable, quadratic equations, completing the square, quadratic formula, square roots, factoring, complex solutions, real numbers, system of equations, variables, systems of linear equations, variables, approximately, graphing linear equations, system of equations, variable, quadratic equation, system of linear equations, matrix, vector variable, coordinate plane, graph, solutions, curve, line, system of linear equations, matrix, vector variable, coordinate plane, graph, solutions, curve, line, x-coordinate, solutions, table of values, polynomial, rational, linear, exponential, logarithmic, absolute value, linear inequality, half-plane, boundary, system of linear inequalities, intersections of half-planes

HS.A-REI.1: Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

Alg 1 Enduring Understandings

Students will know...

1. the definition of equations and solutions

Students will understand...

1. that each step in solving an equation corresponds with an algebraic property

Students will be able to...

1. explain the procedures used when solving an equation

HS.A-REI.2: Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

Alg 2 Enduring Understandings

Students will know...

1. the definition of rational equations, radical equations, and extraneous solution

Students will understand...

1. that extraneous solutions may exist and how to determine if the solution is extraneous

Students will be able to...

1. solve rational and radical equations in one variable

HS.A-REI.3: Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Alg 1 Enduring Understandings

Students will know...

1. the definition of linear equations, coefficients, inequalities

Students will understand...

1. that regardless of the coefficient, the same procedures are used in solving linear equations and inequalities

Students will be able to...

1. solve linear equations and inequalities independent of the type of coefficient

HS.A-REI.4: Solve quadratic equations in one variable.		
a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.		
b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b .		
Alg 2 Enduring Understandings		
<i>Students will know...</i> 1. the definition of variable, quadratic equations, completing the square, quadratic formula, square roots, factoring, complex numbers, complex solutions, real numbers	<i>Students will understand...</i> 1. that the method used is independent of the solution of the equation	<i>Students will be able to...</i> 1. solve quadratic equation using several different methods
HS.A-REI.5: Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.		
Alg 1 Enduring Understandings		
<i>Students will know...</i> 1. the definition of system of equations, variables and a solution of a system of equations	<i>Students will understand...</i> 1. that by adding a multiple of one of the equations to the other equation will produce a system with the same solution that is the same as the original system (elimination/addition method)	<i>Students will be able to...</i> 1. solve a system of equations in two variables
HS.A-REI.6: Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.		
Alg 1 Enduring Understandings		
<i>Students will know...</i> 1. the definition of systems of linear equations, variables, and approximately	<i>Students will understand...</i> 1. that the intersection of the lines graphed is the solution to the system of equations and the solution may be an approximation	<i>Students will be able to...</i> 1. graph a of systems of equations and determine a solution

HS.A-REI.7: Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. <i>For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$.</i>		
Alg 2 Enduring Understandings		
<i>Students will know...</i> 1. the definition of system of equations, variable, and quadratic equation	<i>Students will understand...</i> 1. that on a graph the intersection(s) of a linear equation and a quadratic equation is the solution to the system	<i>Students will be able to...</i> 1. solve a system of equations graphically or algebraically
HS.A-REI.8: (+) Represent a system of linear equations as a single matrix equation in a vector variable.		
Alg 3 Enduring Understandings		
<i>Students will know...</i> 1. the definition of system of linear equations, matrix, vector variable	<i>Students will understand...</i> 1. that a system of equations can be represented and solved using a matrices	<i>Students will be able to...</i> 1. use matrices to solve a system of equations in 2 or more variables
HS.A-REI.9: (+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3×3 or greater).		
Alg 3 Enduring Understandings		
<i>Students will know...</i> 1. the definition of inverse of matrix and system of linear equations	<i>Students will understand...</i> 1. that you can find a solution of a system of equations using the inverse of a matrix	<i>Students will be able to...</i> 1. determine the inverse of a matrix (if it exists) and use the inverse to find the solution to a system of equations; using technology for matrices 3×3 or greater
HS.A-REI.10: Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).		
Alg 1 Enduring Understandings		
<i>Students will know...</i> 1. the definition of coordinate plane, graph, solutions, and curve	<i>Students will understand...</i> 1. that the graph of an equation in two variables is a visual representation of all of the solutions to that equation	<i>Students will be able to...</i> 1. graph an equation in two variables

HS.A-REI.11: Explain why the x -coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

Alg 2 Enduring Understandings

Students will know...

1. the definition of x -coordinate, solutions, table of values, polynomial, rational, linear, exponential, logarithmic, absolute value functions

Students will understand...

1. the intersection(s) of the graphs of functions are the solution(s) to the system of functions

Students will be able to...

1. graph various types of functions

HS.A-REI.12: Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Alg 1 Enduring Understandings

Students will know...

1. the definition of linear inequality, half-plane, boundary, system of linear inequalities, intersections of half-planes

Students will understand...

1. that the solution set to a two variable inequality is a half-plane and the solution set to a system of inequalities is the intersection of half-planes

Students will be able to...

1. graph a system of linear inequalities and show which section of the plane is the designated solution

High School – Seeing Structure in Expressions

Essential Questions:

How do you use patterns to understand mathematics and model situations?

What is algebra?

How are the horizontal and vertical axes related?

How do algebraic representations relate and compare to one another?

How can we communicate and generalize algebraic relationships?

Essential Vocabulary - factor, term, coefficient, expression, factoring, expressions, equivalent expression, quadratic expression, exponential functions

HS.A-SSE.1: Interpret expressions that represent a quantity in terms of its context.

a. Interpret parts of an expression, such as terms, factors, and coefficients.

b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P .*

Alg 1, Alg 2 Enduring Understandings

Students will know...

1. the definition of term, factor, coefficient

Students will understand...

1. that an expression can be viewed as a single term or by its individual parts

Students will be able to...

1. identify the different parts of an expression.

HS.A-SSE.2: Use the structure of an expression to identify ways to rewrite it. *For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.*

Alg 1 Enduring Understandings

Students will know...

1. the definition of equivalency

Students will understand...

1. that rewriting an expression in an equivalent form may allow further manipulation

Students will be able to...

1. rewrite expressions in an equivalent form

HS.A-SSE.3: Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

a. Factor a quadratic expression to reveal the zeros of the function it defines.

b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.

c. Use the properties of exponents to transform expressions for exponential functions. *For example the expression 1.15^t can be rewritten as $(1.15^{1/12})^{12t} \approx 1.01212^t$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*

Alg 1, Alg 2 Enduring Understandings

Students will know...

1. the definition of expressions, equivalency, quadratic expression, and exponential functions

Students will understand...

1. that writing an expression/function in an equivalent form can reveal specific properties of the expression/function

Students will be able to...

1. factor, complete the square, and transform exponential functions

HS.A-SSE.4: Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.
For example, calculate mortgage payments.

Alg 3 Enduring Understandings

Students will know...

1. the formula for a finite geometric series

Students will understand...

1. that the sum of a finite geometric series is dependent upon the first term and the common ratio

Students will be able to...

1. derive the formula for a finite geometric series

DRAFT